Mathematics
Higher level
Paper 3 – calculus

Thursday 15 November 2018 (afternoon)

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.

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Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 10]

(a) Use the limit comparison test to determine whether the series $\sum_{n=1}^{\infty} \frac{2n+1}{3n^2}$ converges or diverges. [5]

(b) Show that the series $\sum_{n=1}^{\infty} \frac{n^2}{n!}(x-1)^n$ converges for all $x \in \mathbb{R}$. [5]

2. [Maximum mark: 8]

(a) Use L'Hôpital's rule to determine the value of

$$\lim_{x \to 0} \left(\frac{e^{-3x^2} + 3\cos(2x) - 4}{3x^2} \right).$$ [5]

(b) Hence find $\lim_{x \to 0} \left(\frac{\int_0^x \left(e^{-3t^2} + 3\cos(2t) - 4 \right) dt}{\int_0^x 3t^2 \, dt} \right)$. [3]

3. [Maximum mark: 14]

Consider the differential equation

$$(x+2)^2\frac{\mathrm{d}y}{\mathrm{d}x}=(x+1)y\text{ , where } x\neq -2$$

with initial condition $y=2$ when $x=1$.

(a) Show that $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=-\frac{3x+7}{(x+2)^2}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. [5]

Taylor polynomials, about $x=1$, are used to approximate $y(x)$.

(b) Find the Taylor polynomial of

(i) degree 2;

(ii) degree 3. [7]

(c) Find the difference between the approximated values of $y(1.05)$ that is obtained using the two answers to part (b). [2]

4. [Maximum mark: 18]

Consider the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x}=1+\frac{y}{x}$, where $x\neq 0$.

(a) Given that $y(1)=1$, use Euler's method with step length $h=0.25$ to find an approximation for $y(2)$. Give your answer to two significant figures. [4]

(b) Solve the equation $\frac{\mathrm{d}y}{\mathrm{d}x}=1+\frac{y}{x}$ for $y(1)=1$. [6]

(c) Find the percentage error when $y(2)$ is approximated by the final rounded value found in part (a). Give your answer to two significant figures. [3]

Consider the family of curves which satisfy the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x}=1+\frac{y}{x}$, where $x\neq 0$.

(d) (i) Find the equation of the isocline corresponding to $\frac{\mathrm{d}y}{\mathrm{d}x}=k$, where $k\neq 0, k\in\mathbb{R}$.

(ii) Show that such an isocline can never be a normal to any of the family of curves that satisfy the differential equation. [5]